Math: Algebra II

Translations and Scale Changes

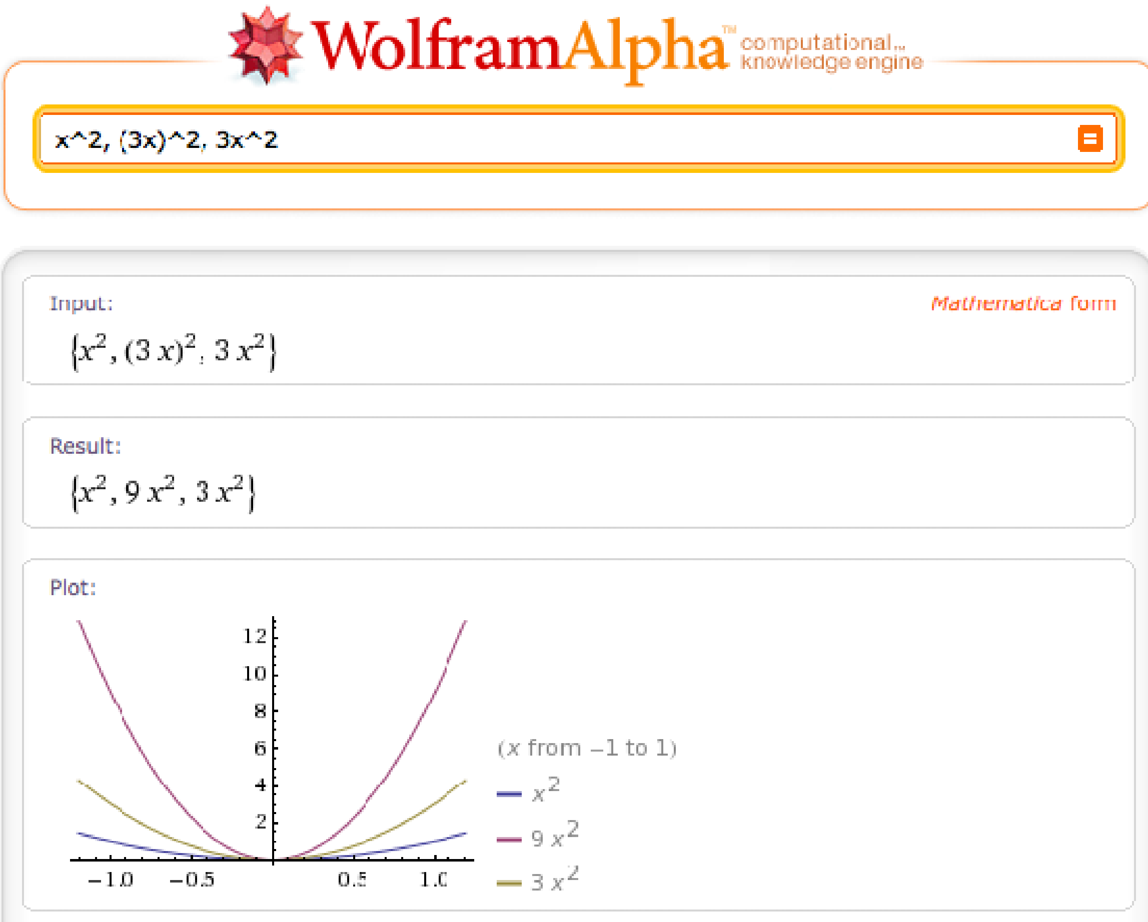
Objectives

Students will be able to:

- Visualize a transformation to a function.
- Relate an original function $f(x)$ to its transformed function.
- Compare function transformations.
- Sketch the graph of a given function.
- Orally explain what a given transformation does to a function.

Warm-Up

What is the difference between the graphs of x^2, $(3x)^2$, and $3x^2$? Use W|A to investigate.



Lesson

• Split students into small groups. The goal is to find how *a*, *b*, *c*, and *d* change the original graph of $f(x)$ when $y = a\,f((x - b)/c)) + d$. Give students access to Wolfram|Alpha and have them research the following functions:

◇ $\cos(x)$

◇ $\cos(10 + x)$

◇ $\cos(x) + 10$

◇ $10\cos(x)$

◇ $\cos(10\,x)$

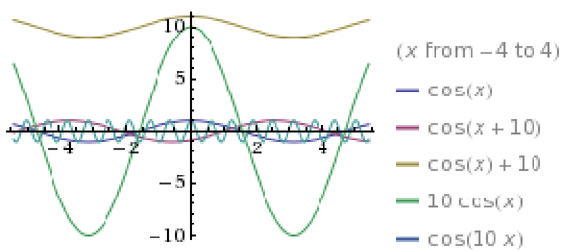
WolframAlpha computational knowledge engine

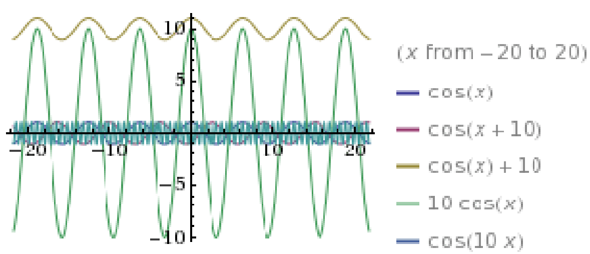
cos(x), cos(x+10), cos(x)+10, 10cos(x), cos(10x)

Input: *Mathematica* form

$\{\cos(x), \cos(x+10), \cos(x)+10, 10\cos(x), \cos(10x)\}$

Plots:





Property:

Periodic in x with period 2π

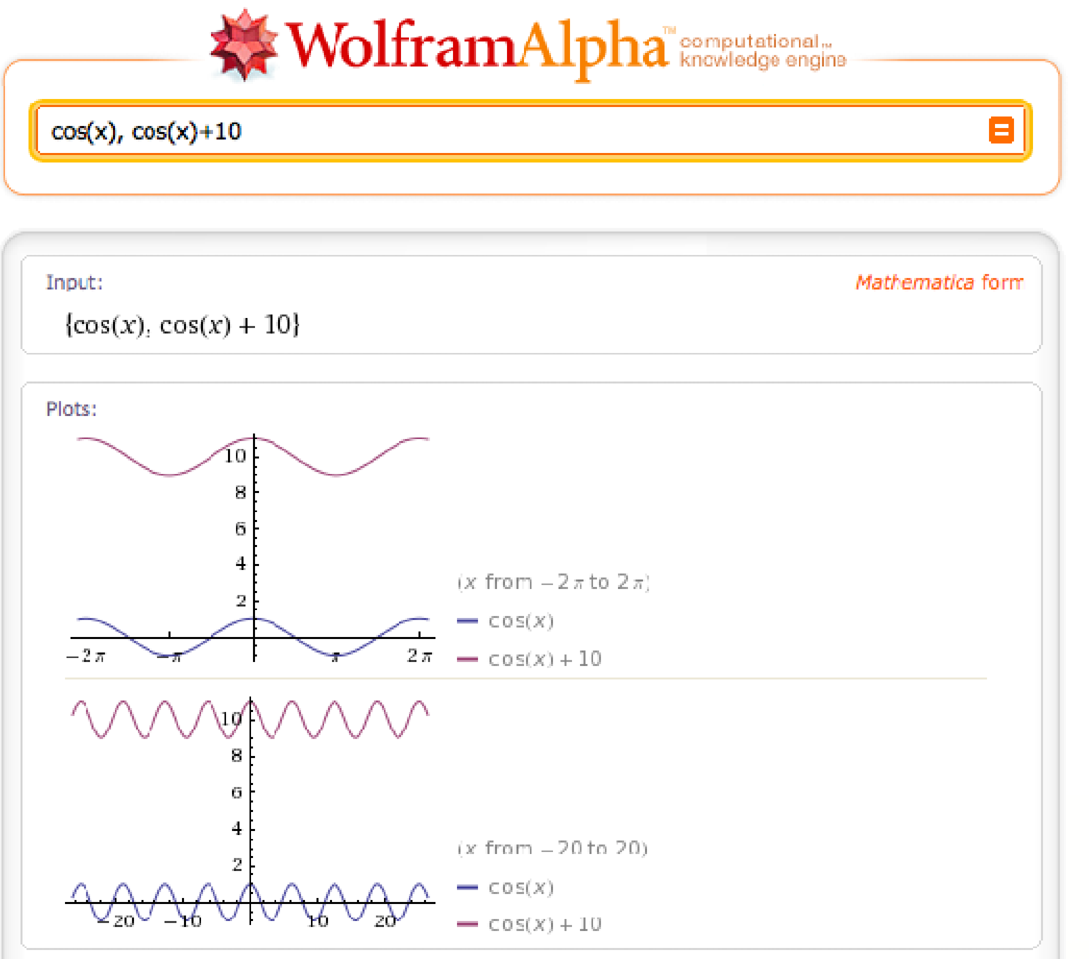
Total:

$$\cos(x) + \cos(10 + x) + (10 + \cos(x)) + 10\cos(x) + \cos(10x) = 12\cos(x) + \cos(10x) + \cos(x + 10) + 10$$

Mean value:

$$\frac{1}{5}(12\cos(x) + \cos(10x) + \cos(x+10) + 10)$$

Computed by: Wolfram *Mathematica* Download as: PDF | Live *Mathematica*



◇ $x^2 + 5$

◇ $(x + 3)^2 + 5$

◇ $\left(x^2 + 5\right) + 3$

◇ $3\left(x^2 + 5\right)$

◇ $(3\,x)^2 + 5$

WolframAlpha computational... knowledge engine

x^2+5, (x+3)^2+5, (x^2+5)+3, 3(x^2+5), (3x)^2+5

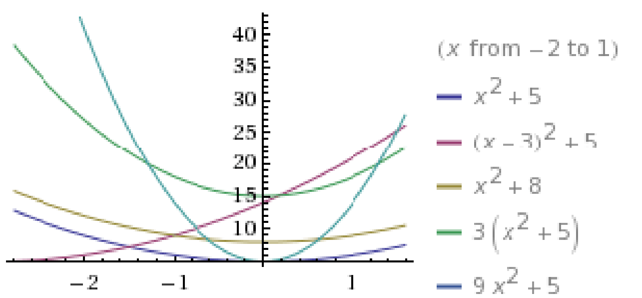
Input: *Mathematica* form

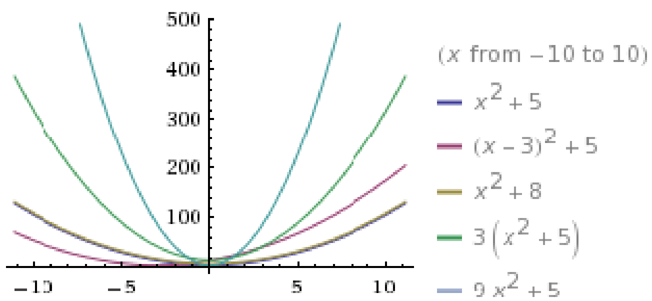
$$\{x^2+5,\ (x+3)^2+5,\ (x^2+5)+3,\ 3\,(x^2+5),\ (3\,x)^2+5\}$$

Result:

$$\{x^2+5,\ (x+3)^2+5,\ x^2+8,\ 3\,(x^2+5),\ 9\,x^2+5\}$$

Plots:





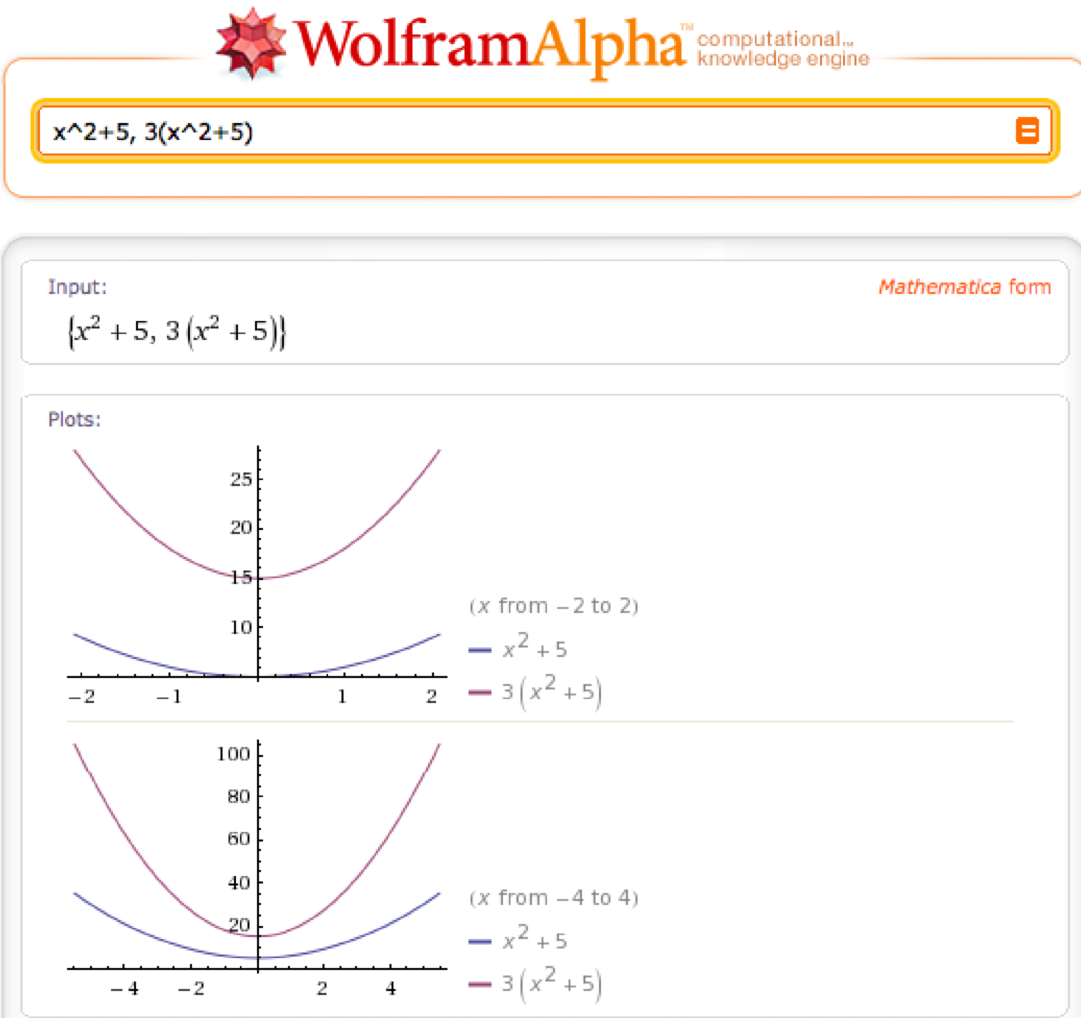
Total:

$$(5+x^2)+(5+(3+x)^2)+(8+x^2)+3\,(5+x^2)+(5+9\,x^2)=$$
$$11\,x^2+3\,(x^2+5)+(x+3)^2+23$$

Mean value:

$$\frac{1}{5}\,(11\,x^2+3\,(x^2+5)+(x+3)^2+23)$$

Computed by: Wolfram *Mathematica* Download as: PDF | Live *Mathematica*



◇ 2^x
◇ $4\left(2^x\right)$
◇ 2^{4x}
◇ $2^{(x+4)}$
◇ 2^x+4

WolframAlpha computational knowledge engine

2^x, 4(2^x), 2^(4x), 2^(x+4), (2^x)+4

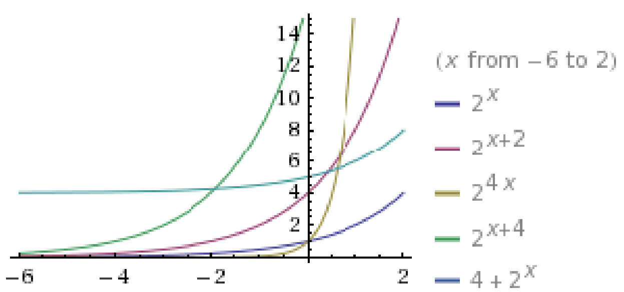
Input: *Mathematica* form

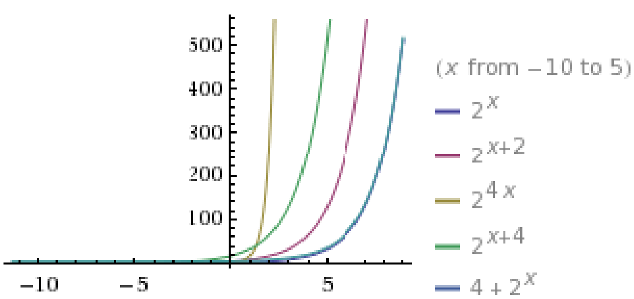
$\{2^x, 4\times 2^x, 2^{4x}, 2^{x+4}, 2^x+4\}$

Result:

$\{2^x, 2^{x+2}, 2^{4x}, 2^{x+4}, 2^x+4\}$

Plots:





Total:

$$2^x + 2^{2+x} + 2^{4x} + 2^{4+x} + (4 + 2^x) = 2^{4x} + 2^{x+1} + 2^{x+2} + 2^{x+4} + 4$$

Mean value:

$$\frac{1}{5}\left(2^{4x} + 2^{x+1} + 2^{x+2} + 2^{x-4} + 4\right)$$

Computed by: Wolfram *Mathematica* Download as: PDF | Live *Mathematica*

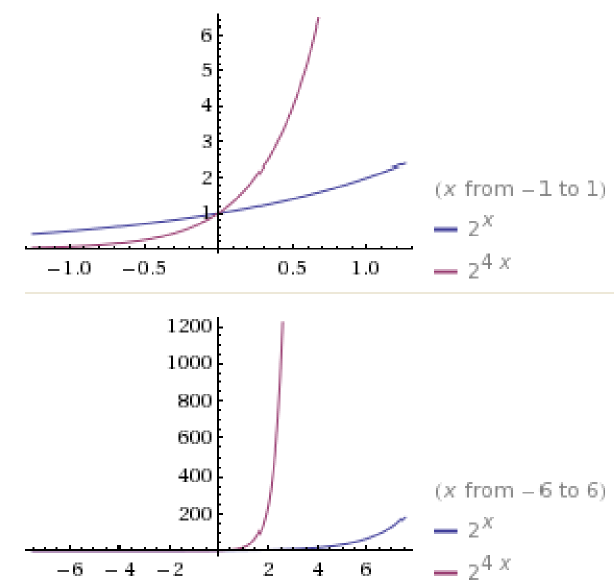
WolframAlpha computational knowledge engine

2^x, 2^(4x)

Input: *Mathematica* form

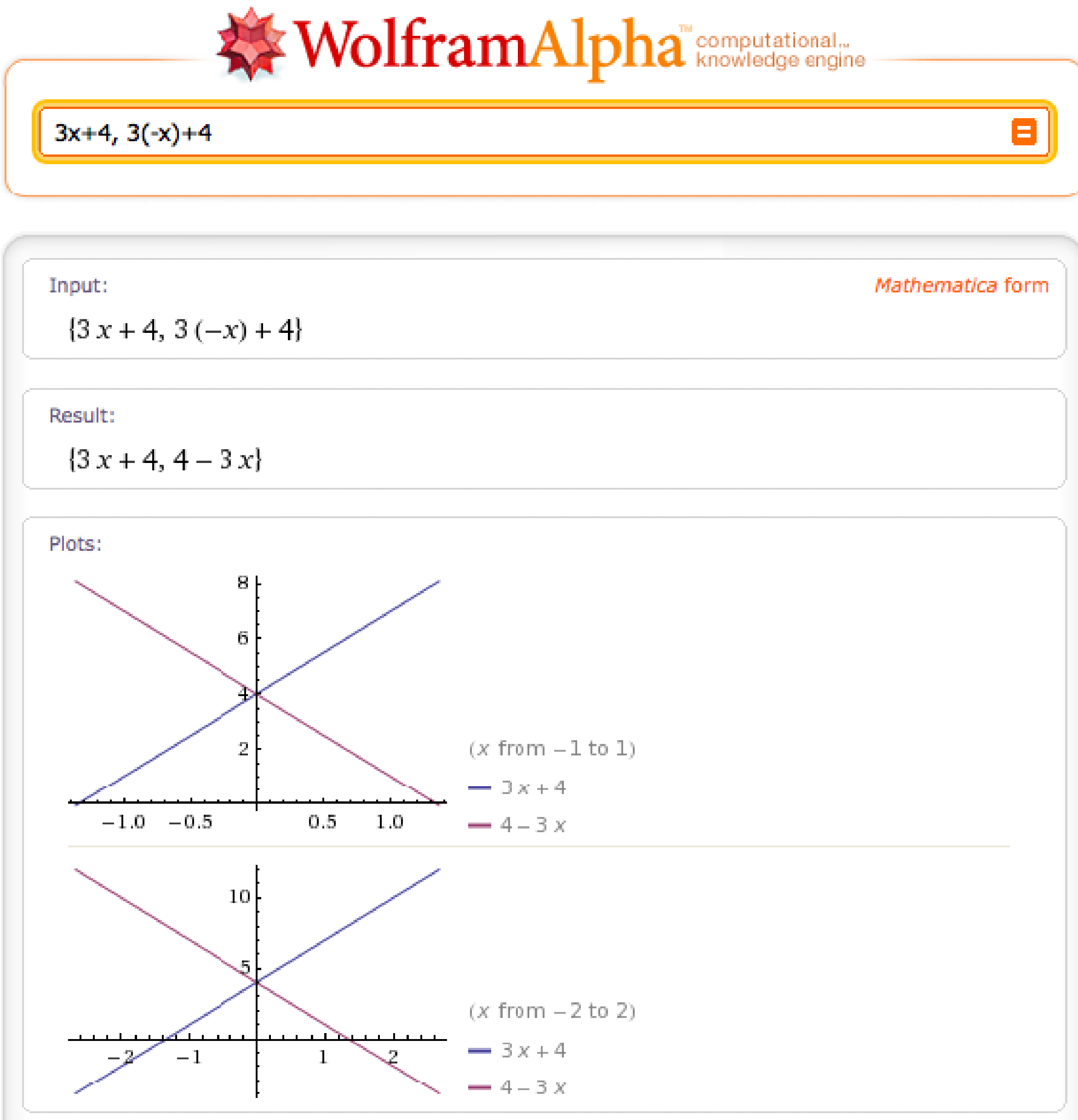
$$\{2^x, 2^{4x}\}$$

Plots:



Note: Students can also use different numbers to investigate the change in the graph.

• Have students come up with a list of proposed suggestions for what *a*, *b*, *c*, and *d* do to the original graph of $f(x)$. Take turns calling on different groups for each variable.
• After each group has contributed, go through what transformations are associated with *a*, *b*, *c*, and *d*.
• What happens when $x \to -x$ or $y \to -y$?

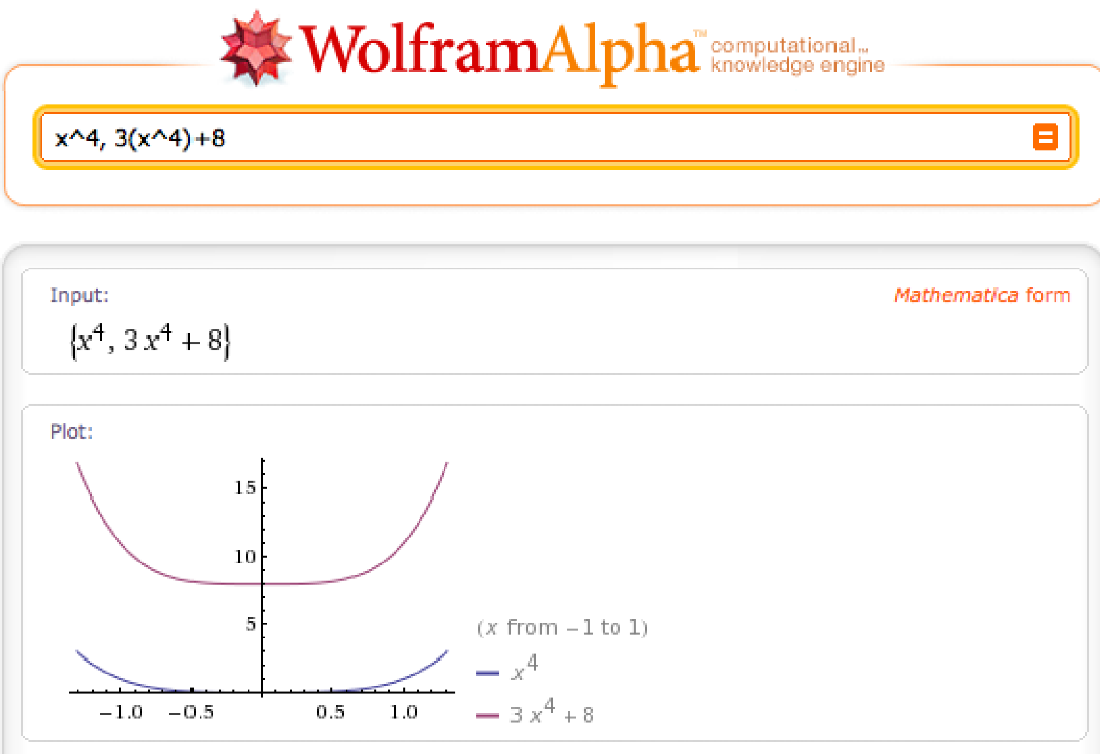


Closing

Let students choose their own function and apply two of the four transformations, where $a = 3$, $b = 5$, $c = 2$, and $d = 8$. Plot the original function as well as the transformed function and explain the transformations.

- $f(x) = x^4$, $a = 3$, $d = 8$

  New function: $g(x) = 3x^4 + 8$



Demonstrations

`Function Transformations`