Objectives

Students will be able to:
• Visualize a transformation to a function.
• Relate an original function \( f(x) \) to its transformed function.
• Compare function transformations.
• Sketch the graph of a given function.
• Orally explain what a given transformation does to a function.

Warm-Up

What is the difference between the graphs of \( x^2 \), \( (3x)^2 \), and \( 3x^2 \)? Use W|A to investigate.
Lesson

• Split students into small groups. The goal is to find how $a$, $b$, $c$, and $d$ change the original graph of $f(x)$ when $y = a f((x - b) / c)) + d$. Give students access to Wolfram|Alpha and have them research the following functions:
  ◊ $\cos(x)$
  ◊ $\cos(10 + x)$
  ◊ $\cos(x) + 10$
  ◊ $10 \cos(x)$
  ◊ $\cos(10 \cdot x)$
Input:

\{\cos(x), \cos(x + 10), \cos(x) + 10, 10 \cos(x), \cos(10x)\}

Property:

Periodic in \(x\) with period \(2\pi\)

Total:

\[\cos(x) + \cos(10 + x) + (10 + \cos(x)) + 10 \cos(x) + \cos(10x) =\]
\[12 \cos(x) + \cos(10x) + \cos(x + 10) + 10\]

Mean value:

\[\frac{1}{5} \left(12 \cos(x) + \cos(10x) + \cos(x + 10) + 10\right)\]
Math: Algebra II Translations and Scale Changes

\[
\cos(x), \cos(x) + 10
\]

- \(x^2 + 5\)
- \((x + 3)^2 + 5\)
- \((x^2 + 5) + 3\)
- \(3(x^2 + 5)\)
- \((3x)^2 + 5\)
Math: Algebra II Translations and Scale Changes

Input:
\[ \{ x^2 + 5, (x + 3)^2 + 5, (x^2 + 5) + 3, 3(x^2 + 5), (3x)^2 + 5 \} \]

Result:
\[ \{ x^2 + 5, (x + 3)^2 + 5, x^2 + 8, 3(x^2 + 5), 9x^2 + 5 \} \]

Plots:

(x from -2 to 1)
- \( x^2 + 5 \)
- \( (x - 3)^2 + 5 \)
- \( x^2 + 8 \)
- \( 3(x^2 + 5) \)
- \( 9x^2 + 5 \)

(x from -10 to 10)
- \( x^2 + 5 \)
- \( (x - 3)^2 + 5 \)
- \( x^2 + 8 \)
- \( 3(x^2 + 5) \)
- \( 9x^2 + 5 \)

Total:
\[
(5 + x^2) + (5 + (3 + x)^2) + (8 + x^2) + 3(5 + x^2) + (5 - 9x^2) = 11x^2 + 3(x^2 + 5) + (x + 3)^2 + 23
\]

Mean value:
\[
\frac{1}{5} \left( 11x^2 + 3(x^2 + 5) + (x + 3)^2 + 23 \right)
\]
$x^2 + 5, 3(x^2 + 5)$

Plots:

- For $x$ from $-2$ to $2$:
  - $x^2 + 5$
  - $3(x^2 + 5)$

- For $x$ from $-4$ to $4$:
  - $x^2 + 5$
  - $3(x^2 + 5)$

- $2^x$
- $4(2^x)$
- $2^4x$
- $2^{(x+4)}$
- $2^x + 4$
Input:
\[\{2^x, 4 \times 2^x, 2^4x, 2^{x+4}, 2^x + 4\}\]

Result:
\[\{2^x, 2^{x+2}, 2^4x, 2^{x+4}, 2^x + 4\}\]
Note: Students can also use different numbers to investigate the change in the graph.
• Have students come up with a list of proposed suggestions for what $a$, $b$, $c$, and $d$ do to the original graph of $f(x)$. Take turns calling on different groups for each variable.
• After each group has contributed, go through what transformations are associated with $a$, $b$, $c$, and $d$.
• What happens when $x \rightarrow -x$ or $y \rightarrow -y$?
Closing

Let students choose their own function and apply two of the four transformations, where \( a = 3 \), \( b = 5 \), \( c = 2 \), and \( d = 8 \). Plot the original function as well as the transformed function and explain the transformations.

• \( f(x) = x^4, \ a = 3, \ d = 8 \)
  New function: \( g(x) = 3x^4 + 8 \)

Demonstrations

Function Transformations