Math: Precalculus

Quadratic Equations with Complex Coefficients

Objectives

Students will be able to:
- Perform arithmetic operations (addition, subtraction, multiplication, division) on complex numbers.
- Solve quadratic equations with complex coefficients.
- Follow the process of completing the square.
- Recall the quadratic formula.
- Write steps to solving quadratic equations in a logical way.

Warm-Up

Have students try the following arithmetic operations:
- $(-4\,i - 8) + (6\,i - 2)$
- $(2 + 5\,i)\,(-2 + i)$
- $(3 - i)^2$

Check the answers in Wolfram|Alpha.

WolframAlpha computational knowledge engine

(2+5i)(-2+i)

Assuming i is the imaginary unit | Use i as a variable instead

Input: *Mathematica* form

$(2+5\,i)\,(-2+i)$

i is the imaginary unit »

Result:

$-9-8\,i$

Magnitude:

$\sqrt{145} \approx 12.0416$

Phase:

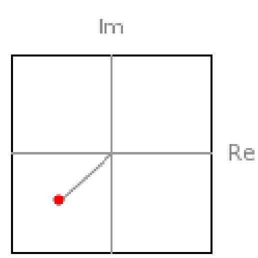
$\tan^{-1}\left(\frac{8}{9}\right)-\pi \approx -138.4°$

$\tan^{-1}(x)$ is the inverse tangent function »

Polar form:

$\sqrt{145}\; e^{i\left(-\pi+\tan^{-1}\left(\frac{8}{9}\right)\right)}$

Position in the complex plane:



WolframAlpha computational knowledge engine

(3-i)^2

Assuming i is the imaginary unit | Use i as a variable instead

Input: *Mathematica* form

$(3-i)^2$

i is the imaginary unit »

Result:

$8-6i$

Magnitude:

10

Phase:

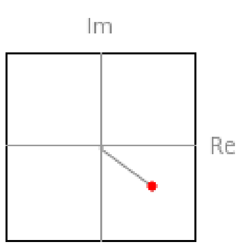
$-\tan^{-1}\left(\frac{3}{4}\right) \approx -36.87°$

$\tan^{-1}(x)$ is the inverse tangent function »

Polar form:

$10\, e^{i\left(-\tan^{-1}\left(\frac{3}{4}\right)\right)}$

Position in the complex plane:



Lesson

• Review completing the square and the quadratic formula. Explain that both methods can be used to solve quadratics with complex coefficients.

Note: Since students have already learned the process with real coefficients, the process should be quick to pick up.

W|A can solve the quadratic formula using:

WolframAlpha computational knowledge engine

quadratic formula

Assuming "quadratic formula" refers to a formula | Use as a function property instead

Calculate indeterminate

- quadratic coefficient: 1
- linear coefficient: -2
- constant coefficient: 1

Input interpretation:

quadratic formula

Equation:

$$x^2 - 2x + 1 = 0$$

$$a x^2 + b x + c = 0$$

a	quadratic coefficient
x	indeterminate
b	linear coefficient
c	constant coefficient

$$\left(x_{\frac{1}{2}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

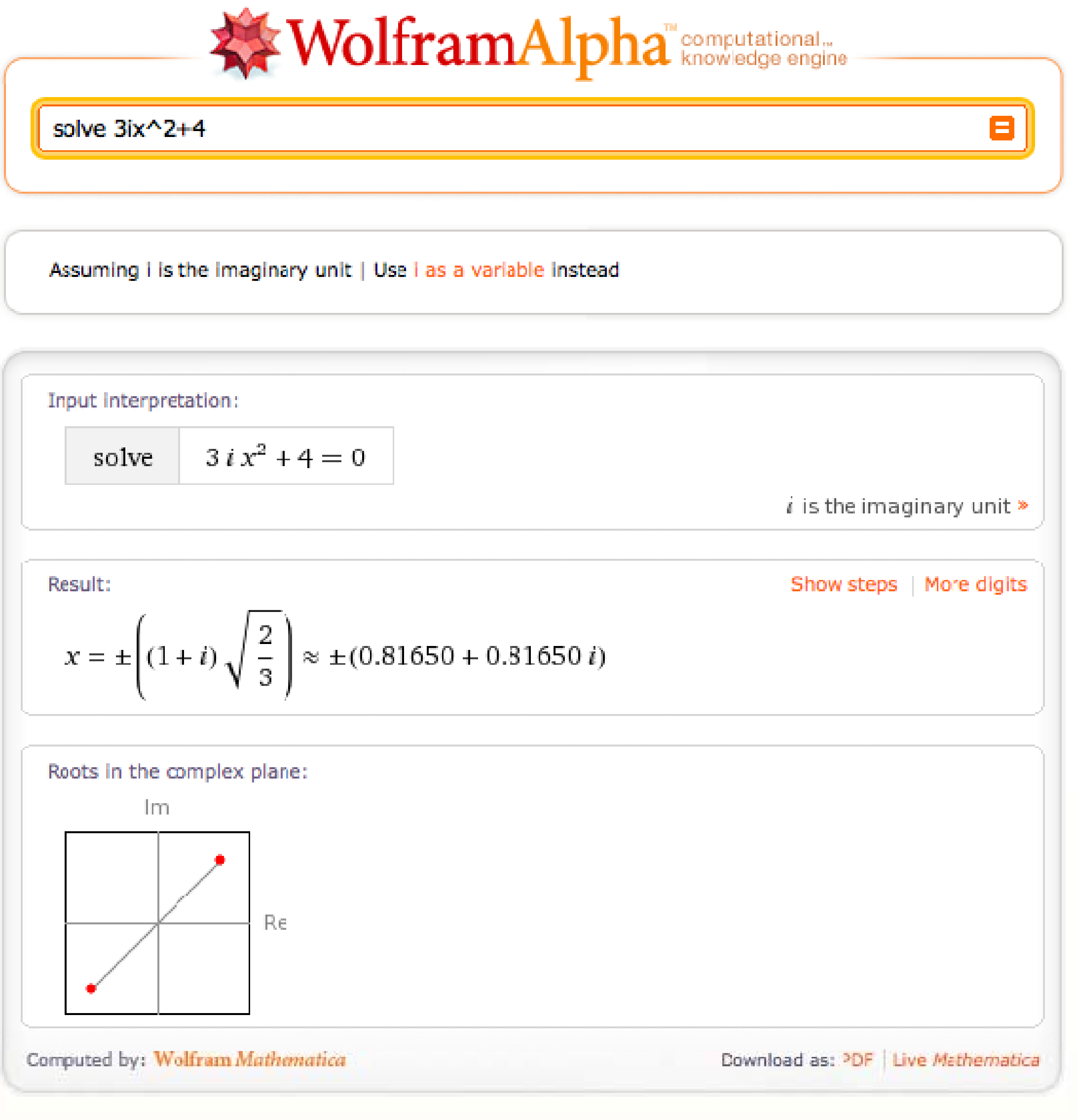
Input values:

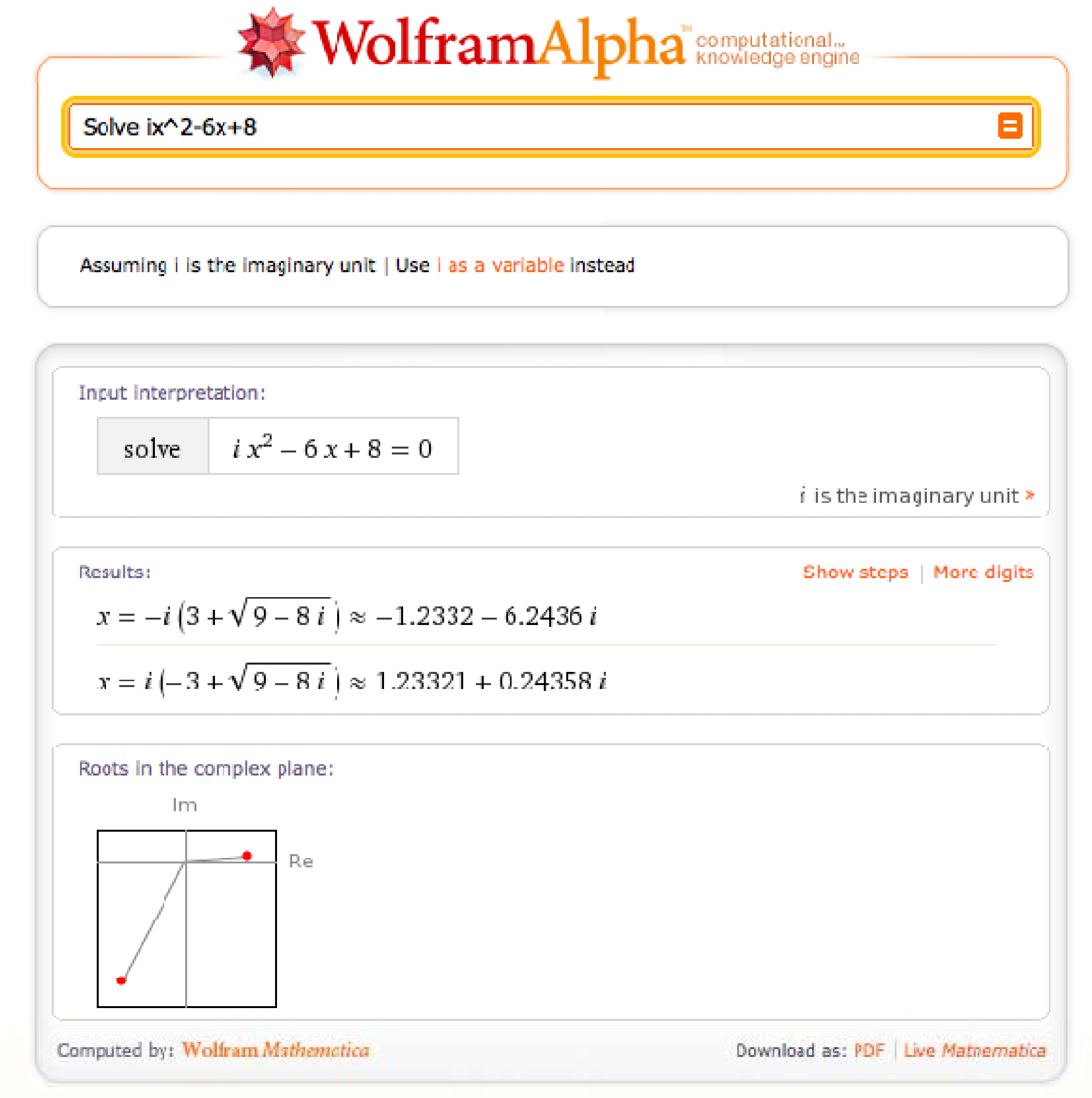
quadratic coefficient	1
linear coefficient	−2
constant coefficient	1

• Have students try basic problems using the method of their choice. They can check their own work using Wolfram|Alpha. The show steps button will help them go through their work.

◇ Solve $3\,i\,x^2 + 4 = 0$.

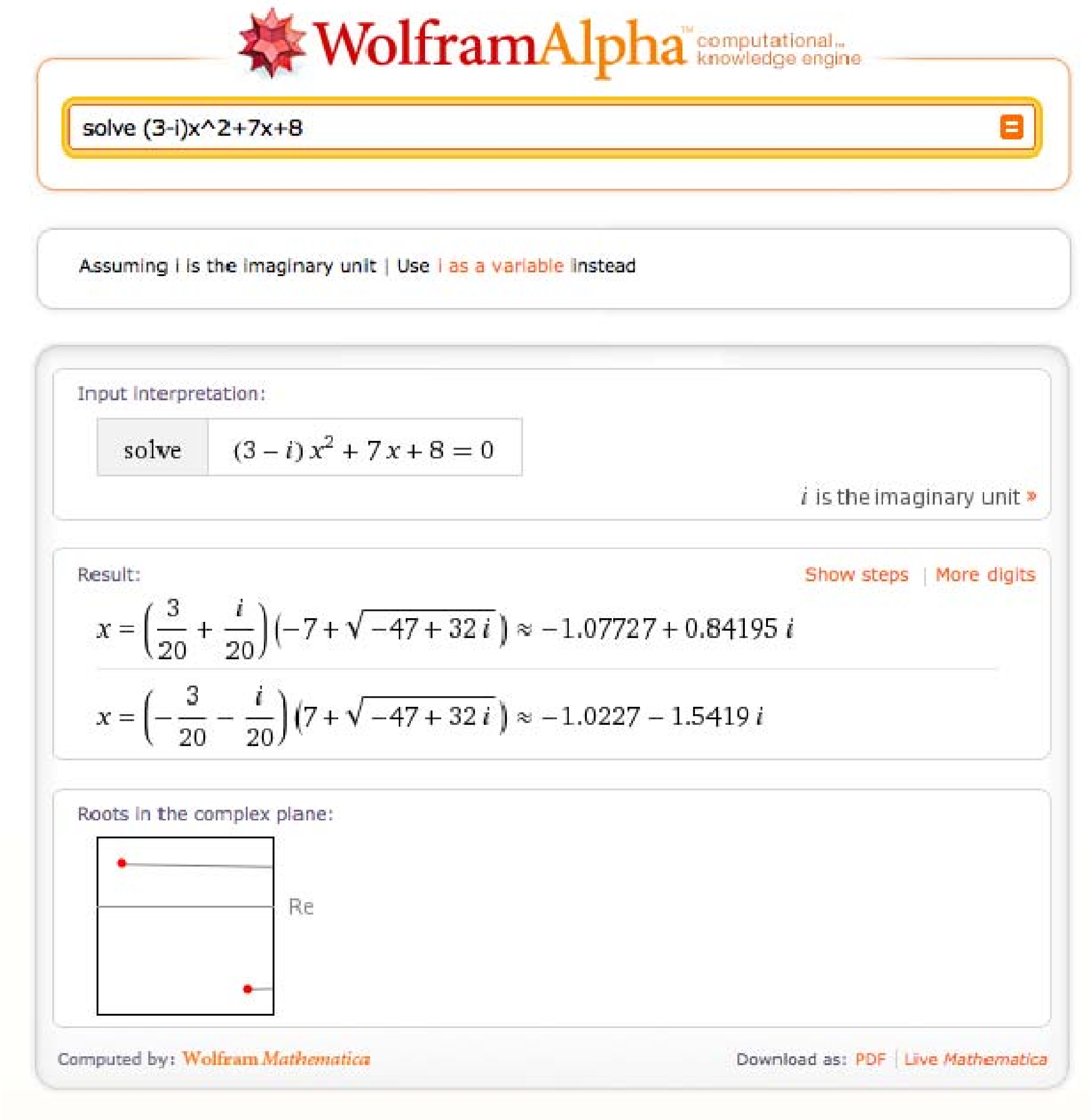
◇ Solve $i\,x^2 - 6\,x + 8 = 0$.





Closing

Test the teacher: Have students give you a complex quadratic to solve, go through the answer on the board, and then prove your answer is correct using Wolfram|Alpha.



Demonstrations

- Completing the Square
- Solution of Quadratic Equations
- Multiplying Complex Numbers
- Polynomial Roots in the Complex Plane