

Math: Precalculus

Operations on Matrices

Objectives

Students will be able to:

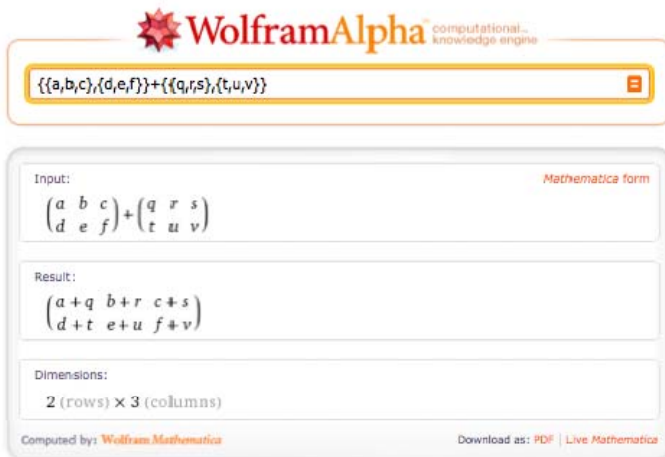
- Perform operations on matrices (addition, scalar multiplication, and matrix multiplication).

Warm-Up

Review what students know about matrices. Set one matrix addition, one scalar multiplication, and one matrix multiplication problem before them and ask students to brainstorm how they might go about solving these problems.

Lesson

- Show students the formula for matrix addition using W|A.



The screenshot shows the WolframAlpha interface. At the top, the WolframAlpha logo is displayed with the tagline "computational... knowledge engine". Below the logo is a search bar containing the input $\{\{a,b,c\},\{d,e,f\}\}+\{\{q,r,s\},\{t,u,v\}\}$. Below the search bar, the input is shown in Mathematica form as $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} q & r & s \\ t & u & v \end{pmatrix}$. The result is shown as $\begin{pmatrix} a+q & b+r & c+s \\ d+t & e+u & f+v \end{pmatrix}$. Below the result, the dimensions are given as "2 (rows) x 3 (columns)". At the bottom, it says "Computed by: Wolfram|Mathematica" and "Download as: PDF | Live Mathematics".

- Have students solve $\begin{pmatrix} 2 & 4 & -1 \\ 0 & 8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 4 \\ 7 & -11 & 2 \end{pmatrix}$ by hand, then check it with W|A.

WolframAlpha™ computational knowledge engine

$\{\{2,4,-1\},\{0,8,3\}\}+\{\{1,6,4\},\{7,-11,2\}\}$

Input: $\begin{pmatrix} 2 & 4 & -1 \\ 0 & 8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 6 & 4 \\ 7 & -11 & 2 \end{pmatrix}$ Mathematica form

Result: $\begin{pmatrix} 3 & 10 & 3 \\ 7 & -3 & 5 \end{pmatrix}$

Dimensions: 2 (rows) x 3 (columns)

Matrix plot:

	1	2	3	
1	3	10	3	1
2	7	-3	5	2
	1	2	3	

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- Show students the formula for scalar multiplication using W|A.

WolframAlpha™ computational knowledge engine

$a*\{\{l,m,n\},\{o,p,q\},\{r,s,t\}\}$

Input: $a \begin{pmatrix} l & m & n \\ o & p & q \\ r & s & t \end{pmatrix}$ Mathematica form

Result: $\{\{a l, a m, a n\}, \{a o, a p, a q\}, \{a r, a s, a t\}\}$

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WolframAlpha™ computational knowledge engine

$\{\{a l, a m, a n\}, \{a o, a p, a q\}, \{a r, a s, a t\}\}$

Assuming "a" is referring to math | Use "a m" as referring to math instead
Assuming "a" is referring to math | Use "a s" as referring to math instead

Input: $\begin{pmatrix} a l & a m & a n \\ a o & a p & a q \\ a r & a s & a t \end{pmatrix}$ Mathematica form

- Have students solve $-3 \begin{pmatrix} 0 & -2 & 1 \\ 4 & 6 & 7 \\ 1 & -5 & 0 \end{pmatrix}$ by hand, then check it with W|A.

WolframAlpha[™] computational knowledge engine

$-3*\{\{0,-2,1\},\{4,6,7\},\{1,-5,0\}\}$

Input: $-3 \begin{pmatrix} 0 & -2 & 1 \\ 4 & 6 & 7 \\ 1 & -5 & 0 \end{pmatrix}$ Mathematica form

Result: $\{\{0, 6, -3\}, \{-12, -18, -21\}, \{-3, 15, 0\}\}$

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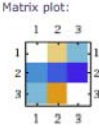
WolframAlpha[™] computational knowledge engine

$\{\{0,6,-3\},\{-12,-18,-21\},\{-3,15,0\}\}$

Input: $\begin{pmatrix} 0 & 6 & -3 \\ -12 & -18 & -21 \\ -3 & 15 & 0 \end{pmatrix}$ Mathematica form

Dimensions: 3 (rows) × 3 (columns)

Matrix plot:



- Show students the formula for matrix multiplication using W|A. Be sure to note that W|A uses a period to represent matrix multiplication.

WolframAlpha[™] computational knowledge engine

$\{\{a,b\},\{c,d\}\}.\{\{e,f\},\{g,h\}\}$

Input: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}.\begin{pmatrix} e & f \\ g & h \end{pmatrix}$ Mathematica form

Result: $\begin{pmatrix} a e + b g & a f + b h \\ c e + d g & c f + d h \end{pmatrix}$

Dimensions: 2 (rows) × 2 (columns)

- Have students solve $\begin{pmatrix} 1 & -1 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}$ by hand, then check using W|A.

WolframAlpha computational knowledge engine

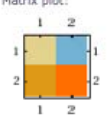
$\{\{1,-1\},\{3,5\}\} \cdot \{\{2,0\},\{1,4\}\}$

Input: $\begin{pmatrix} 1 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}$ Mathematica form

Result: $\begin{pmatrix} 1 & -4 \\ 11 & 20 \end{pmatrix}$

Dimensions: 2 (rows) \times 2 (columns)

Matrix plot:



Closing


- Ask students to solve one or two problems that will help illustrate the constraints on matrix multiplication, that is, the number of columns in the first matrix must equal the number of rows in the second. Make this constraint very explicit for students before they leave.

WolframAlpha computational knowledge engine

$\{\{1,-1,0\},\{3,5,1\}\} \cdot \{\{2,0\},\{1,4\}\}$

Input: $\begin{pmatrix} 1 & -1 & 0 \\ 3 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}$ Mathematica form

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 computational knowledge engine

$\{\{1,-1,0\},\{3,5,1\}\}.\{\{2,0\},\{1,4\},\{1,6\}\}$

Input: Mathematica form


$$\begin{pmatrix} 1 & -1 & 0 \\ 3 & 5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 1 & 4 \\ 1 & 6 \end{pmatrix}$$

Result:

$$\begin{pmatrix} 1 & -4 \\ 12 & 26 \end{pmatrix}$$

Dimensions:
2 (rows) \times 2 (columns)

Matrix plot:



Demonstrations

Matrix Multiplication