

Science: Physics

Gravitational Acceleration

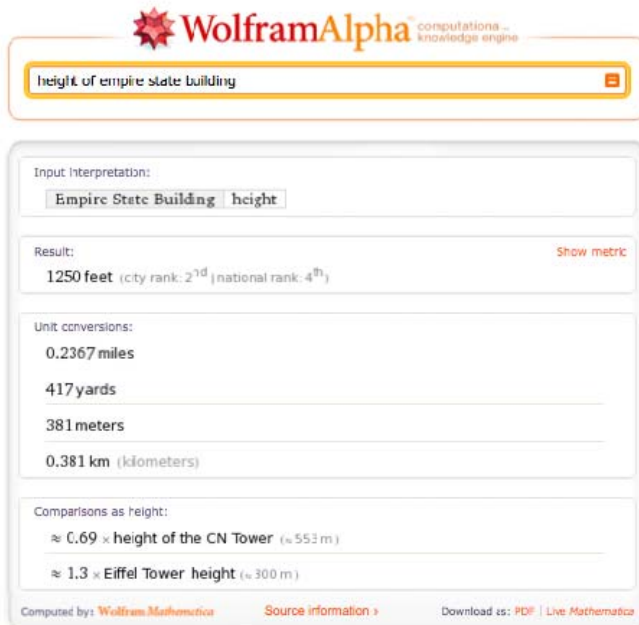
Objectives

Students will be able to:

- Review the formulas for fall time and final velocity of a free-falling object.
- Derive the formula for acceleration due to gravity, g .
- Understand the independence of an object's weight and its acceleration due to gravity.
- Calculate quantifiable effects of air resistance on a falling object.

Warm-Up

Ask students to guess the final velocity of a brick dropped off the roof of the Empire State Building. Use W|A to look up the height of the building.



The screenshot shows the WolframAlpha search interface. The search bar contains the text "height of empire state building". Below the search bar, the input interpretation is "Empire State Building height". The result is "1250 feet (city rank: 2nd | national rank: 4th)". Underneath, unit conversions are listed: "0.2367 miles", "417 yards", "381 meters", and "0.381 km (kilometers)". Comparisons are also provided: "≈ 0.69 × height of the CN Tower (≈ 553 m)" and "≈ 1.3 × Eiffel Tower height (≈ 300 m)". At the bottom, it says "Computed by: Wolfram Mathematica", "Source information", and "Download as: PDF | Live Mathematica".

Then look up the fall time for the object.

WolframAlpha computational knowledge engine

Fall time for an object falling 1250 feet

Assuming no drag Use with drag instead

Input information:

| | |
|---------------------------|-----------|
| object falling vertically | |
| fall distance | 1250 feet |

Result assuming point mass:

| | |
|----------------|-------------------------------|
| time to fall | 8.818 seconds |
| final velocity | 86.42 m/s (meters per second) |

$$t = \sqrt{\frac{2h}{g}}$$


$$v = g t$$

Computed by: Wolfram Mathematica Download as: PDF | Live Mathematica

Now ask students: What would be the final velocity of a block of balsa wood dropped from the same height? Is there a difference? Was mass a variable in the equation used to answer the question?

Lesson

- Review Aristotle's statement that heavy objects fall faster than lightweight ones. Discuss how this statement was accepted as fact for thousands of years, and explain Galileo's overturning of this misconception and Newton's discovery of the mathematical basis for the independence of an object's mass and its gravitational acceleration.
- Ask students to derive for themselves the independence of an object's mass and its gravitational acceleration using Newton's second law and his law of universal gravitation. Check the formulas with W|A.

 computational knowledge engine

Newton's second law

Assuming "Newton's second law" refers to a formula | Use as a word instead

Calculate **force**

■ mass: 100 kg


■ acceleration: 10 m/s²

Input interpretation:
Newton's second law

Equation:

| |
|------------------|
| $F = m a$ |
| F force |
| m mass |
| a acceleration |

(assuming the mass of an object is constant, the net force on an object is equal to its mass multiplied by its acceleration)

 computational knowledge engine

Newton's law of universal gravitation

Calculate **gravitational force**

■ primary mass: 5.98×10^{24} kg

■ secondary mass: 60 kg

■ distance: 6.38×10^6 m

Input interpretation:
Newton's law of universal gravitation

Equation:

| |
|---|
| $F = \frac{G m_1 m_2}{r^2}$ |
| F gravitational force |
| G gravitational constant ($\approx 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2)$) |
| m_1 primary mass |
| m_2 secondary mass |
| r distance |

- Explain that in the case of gravitational acceleration, the term a in the second law is equal to gravitational acceleration g , so the force F must be the force of gravity and the two equations can be set equal to each other. Have students rename m_1 to m and m_2 to M in the formula for gravitational force and solve for a (check solution with W|A).

WolframAlpha computational knowledge engine

Solve $m \cdot a = G \cdot m \cdot M / r^2$ for a

Assuming "m" is referring to math | Use "m*a" as a unit instead

Input Interpretation: Mathematica form
 solve $m a = \frac{G m M}{r^2}$ for a

Results: Show steps
 $m = 0$ and $r \neq 0$
 $a = \frac{G M}{r^2}$ and $r \neq 0$

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- Ask students why Aristotle's error was considered true for so long. Students should recognize that falling objects experience forces of air resistance that *do* depend in part upon mass.

Closing

- Ask students to use W|A to predict the effects of air resistance (drag) on falling objects made of brick and balsa wood. For simplicity's sake, use spheres instead of bricks (W|A can tell you the average brick has dimensions 8x4x2.5, so instruct them to use 8 inches for the diameter of the sphere). Students will need to know the mass, drag coefficient, and projected area of each sphere. Instruct them to calculate the volume of both spheres and the mass of each, finding densities with W|A.

WolframAlpha computational knowledge engine

Volume of a sphere with diameter 8in

Assuming the input refers to a formula | Use "a sphere" as a geometric object instead

Assuming diameter | Use radius instead

Input Information:
 volume of a solid sphere
 diameter 8 inches

Result: More
 volume 4.393 dm³ (cubic decimeters)
 0.004393 m³ (cubic meters)
 4393 cm³ (cubic centimeters)

WolframAlpha computational knowledge engine

4393 (cm)³ * density of brick, 4393 (cm)³ * density of balsa wood

Assuming American balsa wood | Use [DIAB Inc. ProBalsa HW](#) or [more](#) instead
Assuming any type of brick | Use [brick, silica](#) or [more](#) instead

Input Interpretation:
 $\{4393 \times (\text{cm (centimeter)})^3 \times \text{brick density}, 4393 \times (\text{cm (centimeter)})^3 \times \text{American balsa wood density}\}$

Result: 9890 grams | 703 grams [Show details](#)

- Now ask students to assume the drag coefficient of both spheres to be 0.4, and to find their projected surface area (simply the area of a circle with radius 8 in).

WolframAlpha computational knowledge engine

Area of a circle with diameter 8 in

Input Interpretation:
circle diameter 8 inches area

Result: 50.27 in² (square inches)

- Now ask students to input this data in W|A's falling object calculator and find the fall times and final velocities of spheres made of brick and balsa wood.



Fall time

- fall distance:
- release height:
- mass:
- fluid density:
- drag coefficient:
- projected surface area:

Assuming with drag | Use [no drag](#) instead

Input information:

| object falling vertically | |
|---------------------------|--|
| fall distance | 1250 feet |
| release height | 1250 feet |
| mass | 9890 grams |
| fluid density | 1.29 kg/m ³ (kilograms per cubic meter) |
| drag coefficient | 0.4 |
| projected surface area | 50.27 in ² (square inches) |

Result assuming object in viscous medium:

[Show formula](#)

| | |
|---------------------------------------|-------------------------------|
| time to fall | 9.286 seconds |
| slowdown factor | 1.05307 |
| final speed | 74.47 m/s (meters per second) |
| final acceleration | 0.534 g |
| terminal velocity | 109 m/s (meters per second) |
| 99% terminal velocity attainment time | 29.45 seconds |

(assumed medium air)

WolframAlpha computational knowledge engine

Fall time

■ fall distance:
 ■ release height:
 ■ mass:
 ■ fluid density:
 ■ drag coefficient:
 ■ projected surface area:

Assuming with drag | Use **no drag** instead

Input information:

| object falling vertically | |
|---------------------------|--|
| fall distance | 1250 feet |
| release height | 1250 feet |
| mass | 703 grams |
| fluid density | 1.29 kg/m ³ (kilograms per cubic meter) |
| drag coefficient | 0.4 |
| projected surface area | 50.27 in ² (square inches) |

Result assuming object in viscous medium: [Show formula](#)

| | |
|---------------------------------------|-------------------------------|
| time to fall | 15.16 seconds |
| slowdown factor | 1.71941 |
| final speed | 29.07 m/s (meters per second) |
| final acceleration | 0.000145 g |
| terminal velocity | 29.07 m/s (meters per second) |
| 99% terminal velocity attainment time | 7.851 seconds |

(assumed medium air)

- Point out to students that the final speed of the brick sphere is more than twice that of a similarly shaped object made of balsa wood. Assign them to perform similar computations with objects of different sizes or compositions.

Demonstrations

Free Fall on the Solar System Planets and the Moon

Velocity of a Falling Object

Projectile with Air Drag