Math: Algebra II
Graphing Polynomials of Higher Degree

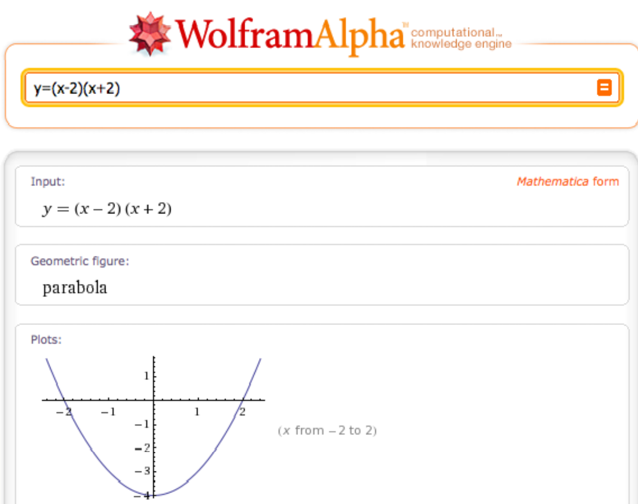
Objectives

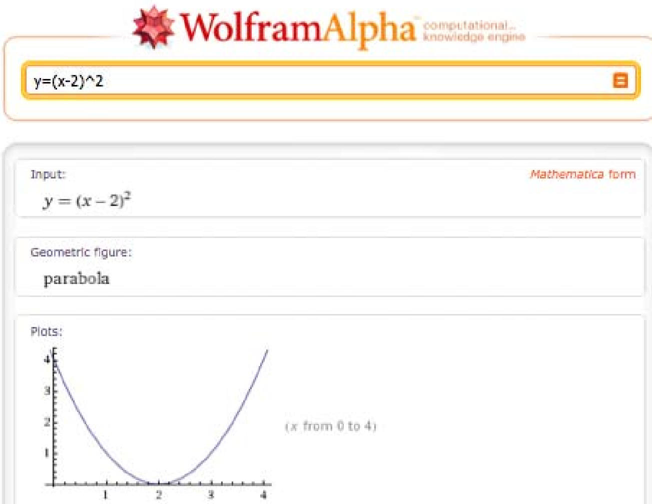
Students will be able to:

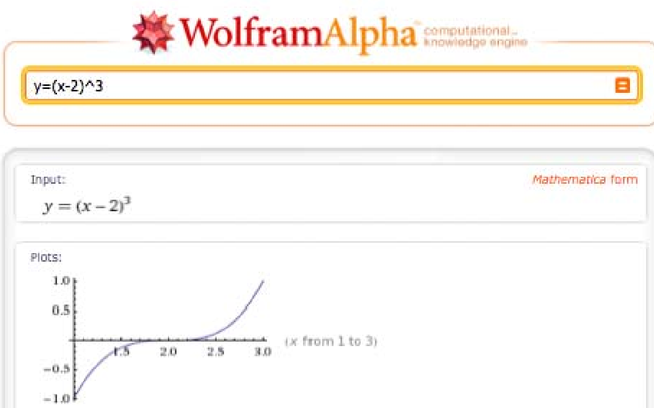
- Relate the real roots of a polynomial to the *x*-intercepts of its graph.
- Graph simple polynomials of degree three and higher.
- Determine possible equations for polynomials of higher degree from their graphs.

Warm-Up

Ask students to sketch graphs of the functions $y = (x-2)(x+2)$, $y = (x-2)^2$, and $y = (x-2)^3$. Check the graphs with W|A.

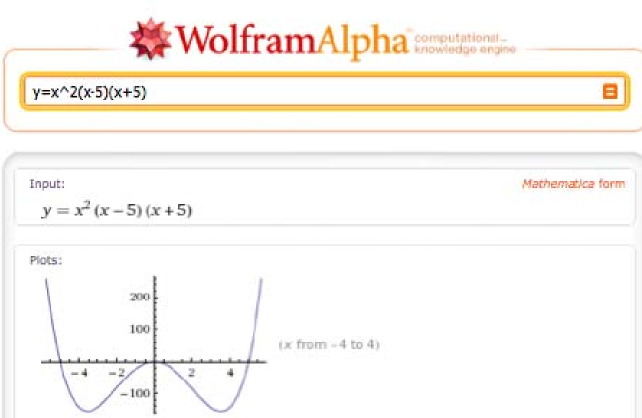




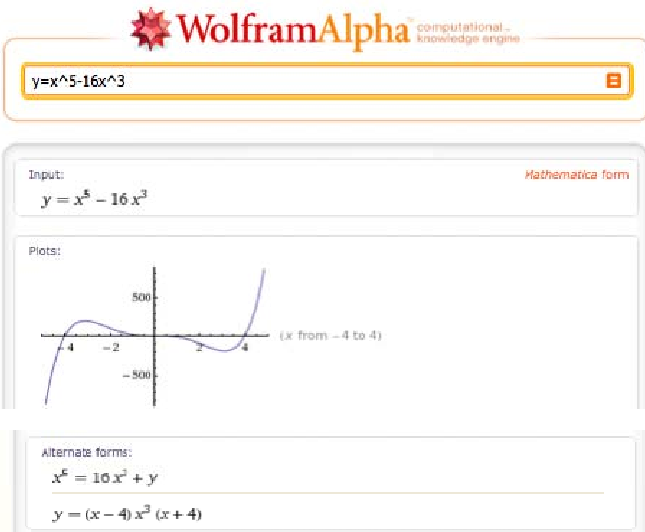


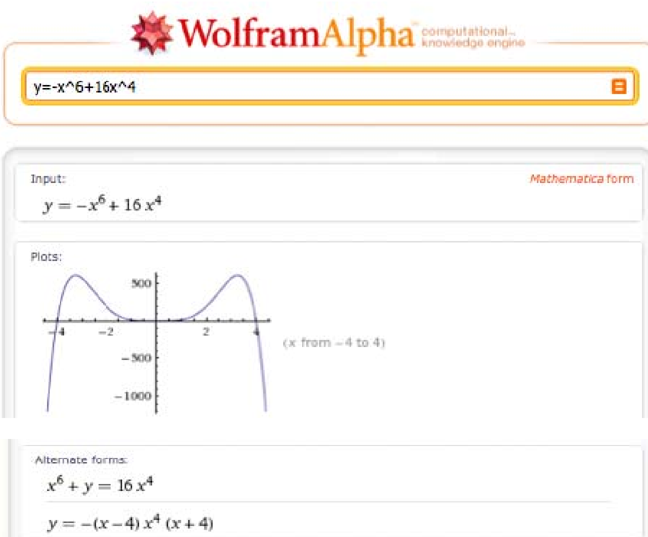
Lesson

• Point out to students that each binomial factor of a polynomial generates a root and that each of these roots is equal to an *x*-intercept of the graph of the polynomial. Ask them to predict the *x*-intercepts of the equation $y = x^2(x-5)(x+5)$ and to check their answers with W|A.

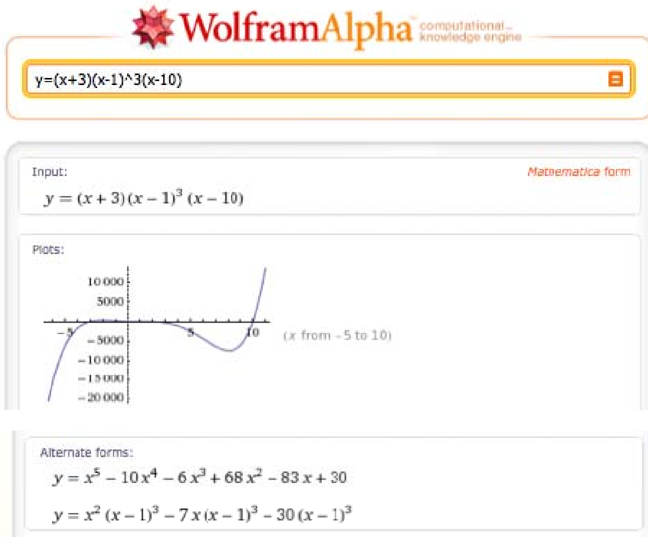


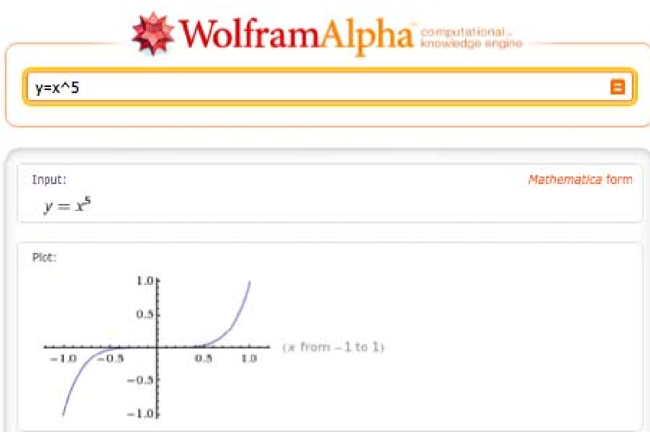
• Explain that the degree of a polynomial and the sign of its leading coefficient affect the general behavior of its graph at increasingly large absolute values of *x*. Illustrate the reflectional and rotational symmetries of even and odd functions respectively with several W|A examples.

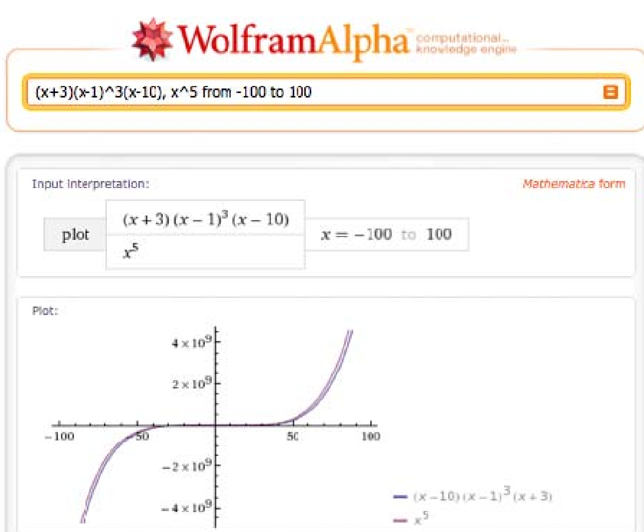




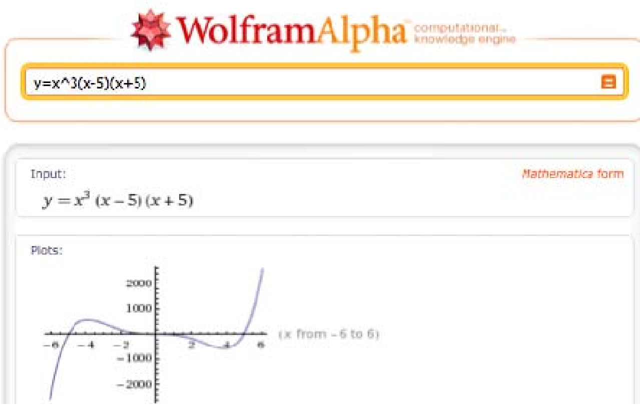
• Point out to students that wherever the absolute value of *x* is large, the numbers in the binomials become insignificant and the polynomial can be approximated by the leading term $a x^n$. For example, $(x+3)(x-1)^3(x-10)$ becomes close to x^5 at large absolute values of *x*.

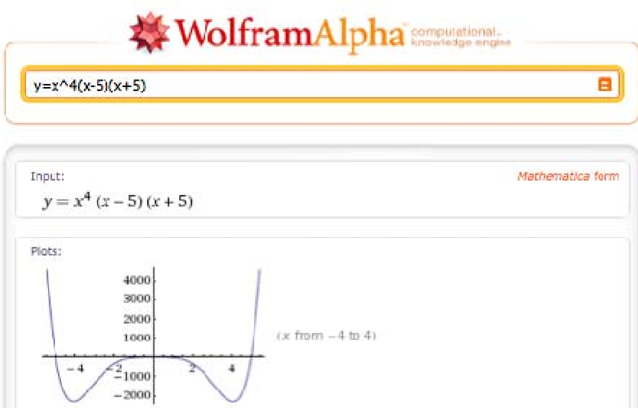

WolframAlpha computational knowledge engine
y=(x+3)(x-1)^3(x-10)
Input:
Mathematica form
$y = (x+3)(x-1)^3(x-10)$
Plots:
(x from -5 to 10)
Alternate forms:
$y = x^5 - 10x^4 - 6x^3 + 68x^2 - 83x + 30$
$y = x^2(x-1)^3 - 7x(x-1)^3 - 30(x-1)^3$

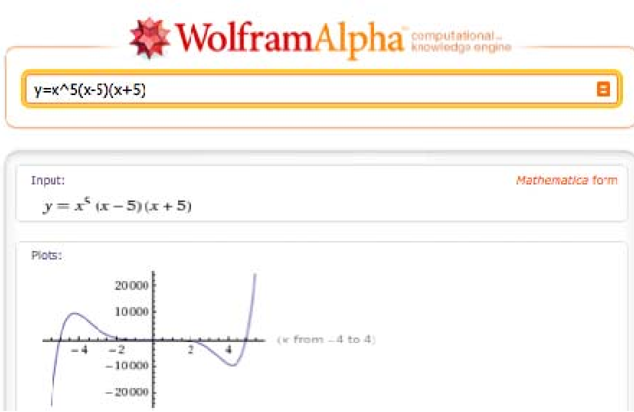

WolframAlpha computational knowledge engine
y=x^5
Input:
Mathematica form
$y = x^5$
Plot:
(x from -1 to 1)


WolframAlpha computational knowledge engine
(x+3)(x-1)^3(x-10), x^5 from -100 to 100
Input interpretation:
Mathematica form
plot
$(x+3)(x-1)^3(x-10)$
x^5
$x = -100$ to 100
Plot:
$(x-10)(x-1)^3(x+3)$
x^5

• Explain to students that the degree of a binomial factor in a polynomial determines the behavior of its graph at the *x*-intercept associated with that binomial factor. Return to the first example, $y = x^2(x - 5)(x + 5)$, and use W|A to illustrate the effects of raising a binomial factor to successively higher powers.

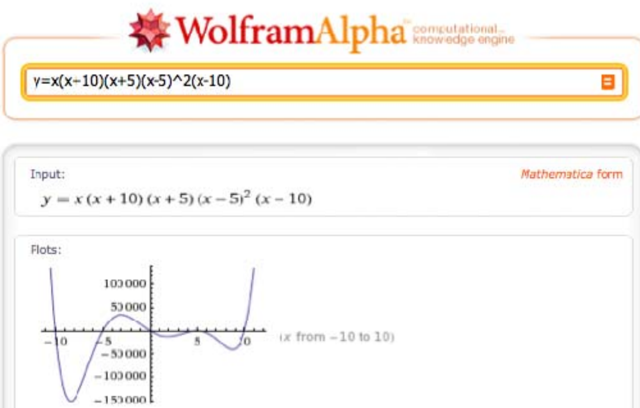






Closing

Use W|A to create a graph of the equation $y = x(x + 10)(x + 5)(x - 5)^2 (x - 10)$ and ask students to derive a possible equation for the polynomial based on the graph and what students have learned during class.



Demonstrations

Polynomial Roots

End Behavior of Polynomial Functions

Local Behavior of a Polynomial Near a Root

Where Are My Roots?

Parameters for Plotting a Cubic Polynomial

Polynomial Graph Generator