

# Math: Calculus I

## Exploring Functions and Their Derivatives

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### Objectives

Students will be able to:

- Graph a function.
- Calculate the derivative of any elementary function.
- Relate and compare a function to its derivative.
- Make assertions about a graph, its derivative, and its higher derivatives.

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### Warm-Up

Find the derivative of  $f(x) = x^2$ . Then sketch the the graphs of the function and derivative together. Take some time to make educated guesses about their relationship.



$x^3$ , derivative of  $x^3$



Input interpretation:

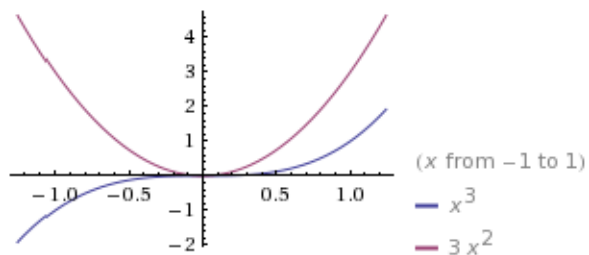
Mathematica form

$$\left\{x^3, \frac{\partial x^3}{\partial x}\right\}$$

Result:

$$\{x^3, 3x^2\}$$

Plot:



## Lesson

- Discuss the relationship between a function and its first derivative.
- ◇ When is the function increasing or decreasing, concave up or concave down?
- ◇ Example: graph  $y = 3x^3 + 4x^2 + 2x + 3$ .

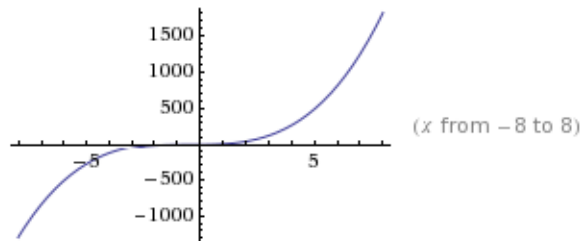
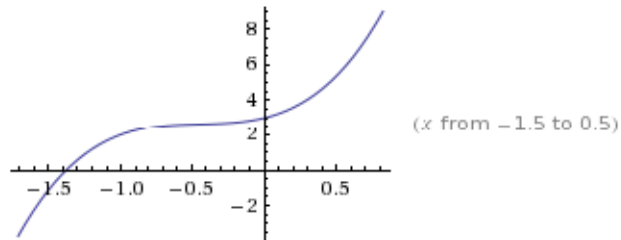

 $3x^3+4x^2+2x+3$ 


Input:

Mathematica form

$$3x^3 + 4x^2 + 2x + 3$$

Plots:



Alternate form:

$$x(x(3x + 4) + 2) + 3$$

Real root:

Exact form

$$x \approx -1.3767$$


Complex roots:

Exact forms

$$x \approx 0.0216846 - 0.851999i$$

$$x \approx 0.0216846 + 0.851999i$$

- ◇ Discuss what the graph of the derivative of this function looks like. Compare the function and its derivative.
- ◇ Have students sketch the derivative of this function using W|A to see if they were correct.


**WolframAlpha**<sup>™</sup> computational...  
 knowledge engine

$3x^3+4x^2+2x+3$ , derivative  $3x^3+4x^2+2x+3$

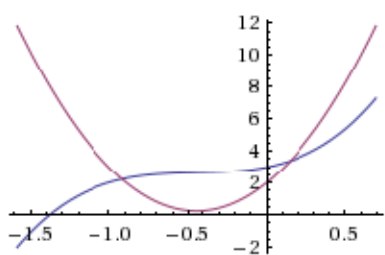
Input interpretation: Mathematica form

$$\left\{ 3x^3 + 4x^2 + 2x + 3, \frac{\partial(3x^3 + 4x^2 + 2x + 3)}{\partial x} \right\}$$

Result:

$$\{ 3x^3 + 4x^2 + 2x + 3, 9x^2 + 8x + 2 \}$$

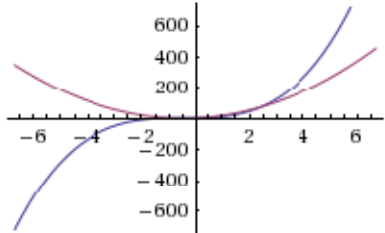
Plots:



(x from -1.5 to 0.5)

- $3x^3 + 4x^2 + 2x + 3$
- $9x^2 + 8x + 2$

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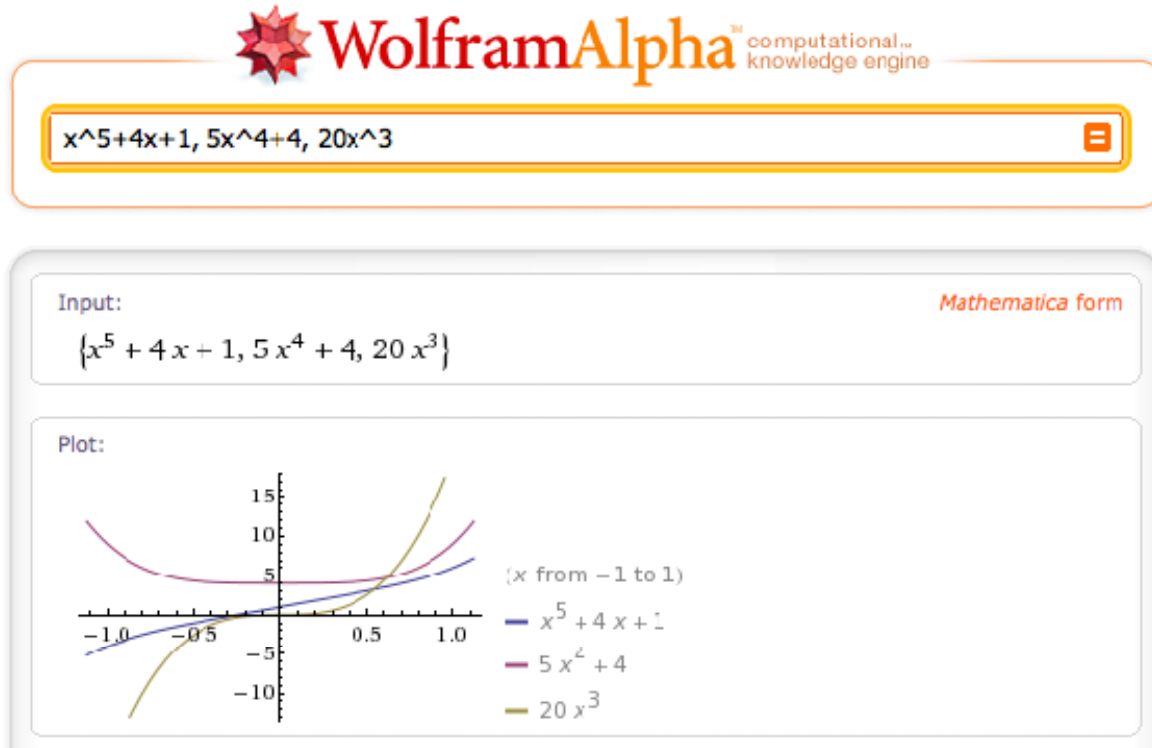


(x from -6 to 6)

- $3x^3 + 4x^2 + 2x + 3$
- $9x^2 + 8x + 2$

- Student examples:

- ◇ Choose a polynomial of degree three or higher.
- ◇ Calculate its first and second derivative.
- ◇ Without using Wolfram|Alpha, try to graph the derivative of your function.
- ◇ Then use Wolfram|Alpha to see the relationship between the two graphs.
- ◇ Graph the function and its first and second derivative.
- ◇ Make conclusions about the three functions on the graph.



- ◇ Choose a function. Find the maximum and minimum on the domain  $[-10, 10]$ , then graph the function as well as its derivative using Wolfram|Alpha.

## Closing

Ticket to leave: If a function is concave up, its derivative is \_\_\_\_\_.

## Demonstrations

Derivatives : A Look at Graphs

Graphing Derivatives

A Fifth-Degree Polynomial and Its Derivatives

Polynomials and Derivatives

Derivative as a Function

Polynomial and Derivative