

Math: Geometry

Calculating Distances in Two and Three Dimensions

Objectives

Students will be able to:

- Apply the Pythagorean theorem to calculate distances in two dimensions.
- Calculate distances between points in three-dimensional space.
- Practice reasoning by analogy.

Warm-Up

Ask students how they might determine the distance between two points in a plane. Prompt them with an example involving a Pythagorean triple: What is the distance between the points $(0, 0)$ and $(3, 4)$?

WolframAlpha computational knowledge engine

distance from $(0,0)$ to $(3,4)$

Input interpretation:

distance

point coordinates $0, 0$

point coordinates $3, 4$

Visual representation:

RESULT:

5

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Lesson

- Point out that plotting two points in the Cartesian plane creates two right triangles sharing a hypotenuse, and that the length of the hypotenuse is the distance between the points.
- Show how the two-dimensional distance formula, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ can be derived from the Pythagorean theorem.
- Have a student give you any pair of points. Calculate the distance between them with the distance formula and then check your solution using W|A.

WolframAlpha computational knowledge engine

distance from (-14,3) to (5,-12)

Input interpretation:

distance

point coordinates -14, 3

point coordinates 5, -12


Visual representation:

Result:

$\sqrt{586} \approx 24.2074$

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- Ask students to consider how they might find the distance between points in three-dimensional space. Illustrate how the Pythagorean theorem can be extended into three dimensions by creating a right triangle with two of its vertices lying in a horizontal plane (see the *Mathematica* Demonstrations at the end of this lesson for a visualization). The result is the three-dimensional distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- Have a student give you any two points in three-dimensional space and calculate the distance between them using W|A.

 computational knowledge engine

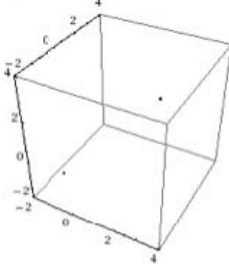
distance (-1,-1,-1) to (2,2,2)

Input interpretation:

distance point coordinates -1, -1, -1

point coordinates 2, 2, 2

Visual representation:



Result:

$3\sqrt{3} \approx 5.19615$

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- Ask students how to determine the distance between two points in one-dimensional space, that is, the number line. The answer is the absolute value of the difference between the points' coordinates. Point out that this comes from removing the y terms from the two-dimensional distance formula to get $\sqrt{(x_2 - x_1)^2}$.

Closing

Ticket to leave: Ask students to study the similarities among the distance formulas for one, two, and three dimensions. Can they think by analogy and guess a formula for the distance between two points in four dimensions? Have students test their guesses by making up two four-dimensional points, using their guesses to calculate the distance between points by hand, and then checking their solutions with W|A.

Demonstrations

Distance between Two Points

Pythagorean Theorem 3 D