Math: Calculus

Absolute Maximum and Minimum

Objectives

Students will be able to:
• Interpret the graph of a function.
• Relate the process of finding maxima and minima to the graphs themselves.
• Find the absolute maximum and minimum of a function on different domains.

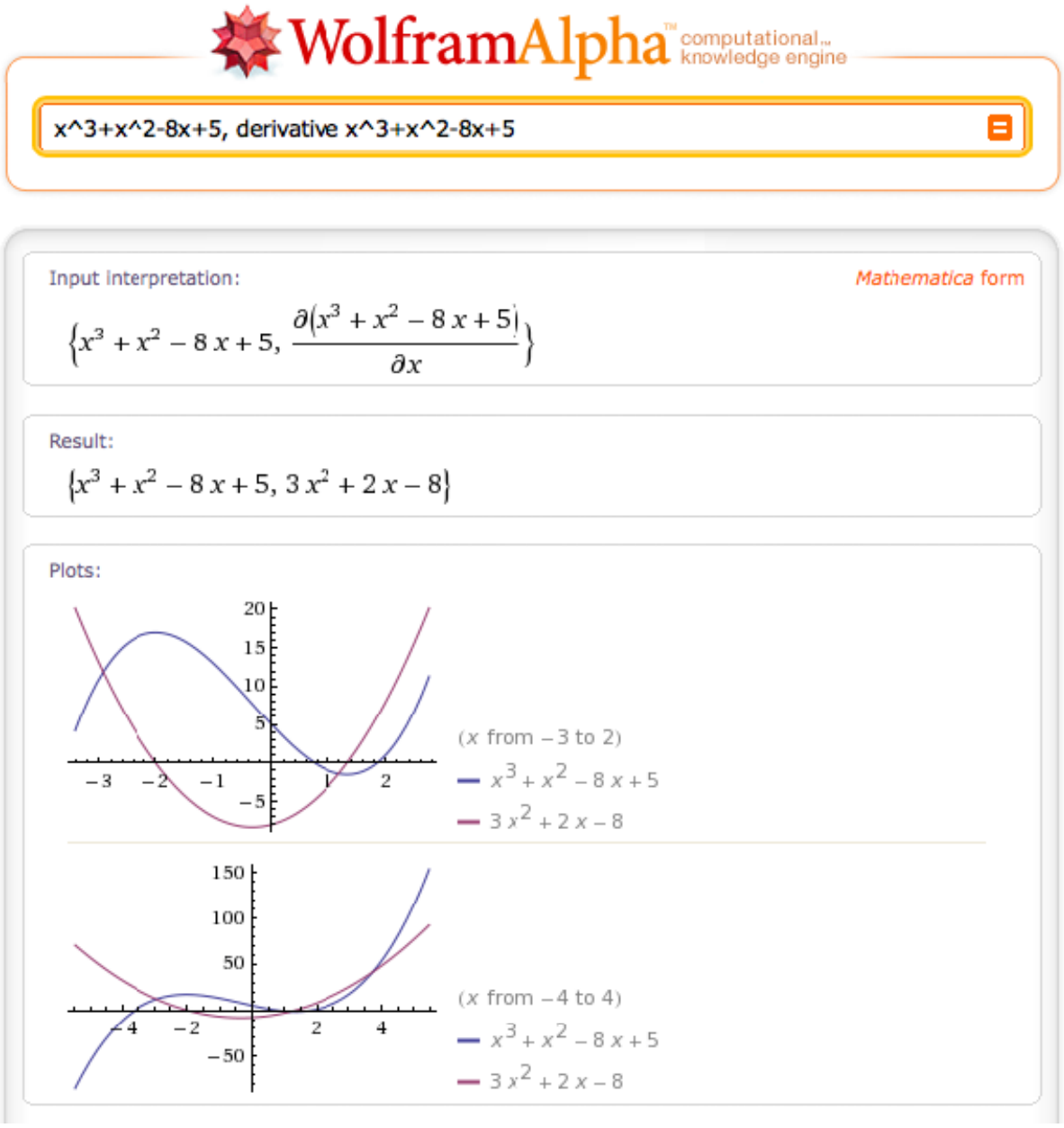
Warm-Up

After having discussed the process of finding the absolute maximum and minimum by finding the derivative and testing at the endpoints yesterday, the goal today is to understand why this process is used. Have students take five minutes to write down an educated guess as to why we set the derivative equal to zero in order to find maxima and minima.

Lesson

• Explain and reinforce the idea that the derivative is the slope of the tangent line, so that it is logical that the derivative is zero at a maximum or minimum.

• Student activity: Graph the following pairs of functions using W|A.
◇ $f(x) = x^3 + x^2 - 8x + 5,\ f'(x)$
◇ $f(x) = \sin(x),\ f'(x)$
◇ $f(x) = 3x^3 - 2x,\ f'(x)$
Make a logical guess as to what x values give the maxima and minima of each function. At those x values, what is the value of $f'(x)$?



Now use the methods learned to find the absolute maximum and minimum on the domain $[-5, 5]$ for each of the three functions.

• Use Wolfram|Alpha to check your answers.

WolframAlpha computational... knowledge engine

maximum x^3+x^2-8x+5, x from -5 to 5

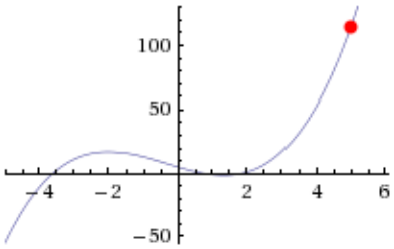
Input interpretation: *Mathematica* form

maximize	function	$5 - 8\,x + x^2 + x^3$
	domain	$-5 \le x \le 5$

Maximum between -5 and 5:

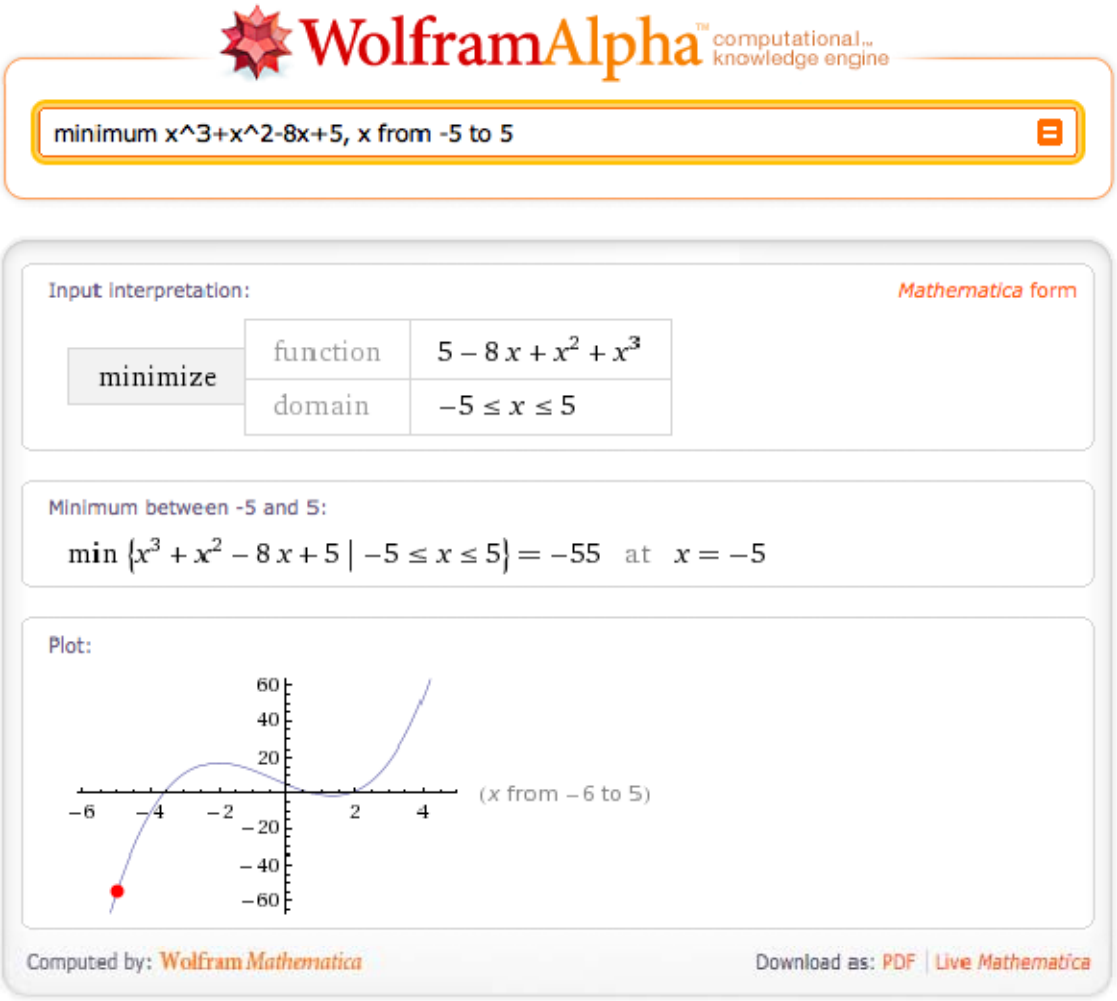
$$\max\left\{x^3 + x^2 - 8\,x + 5 \mid -5 \le x \le 5\right\} = 115 \quad \text{at} \quad x = 5$$

Plot:



(x from -5 to 6)

Computed by: Wolfram *Mathematica* Download as: PDF | Live *Mathematica*



• Tell students to choose a function. Find the maximum and minimum on the domain [-10, 10], and graph the function as well as its derivative using Wolfram|Alpha.

Closing

Fill in the blank of this sentence on a piece of paper. When a function has a maximum or minimum on an infinite domain, the derivative is _____.

Demonstrations

A Fifth-Degree Polynomial and Its Derivatives

Derivative as a Function

Instantaneous Rate of Change: Exploring More Functions with the First and Second Derivatives

Tangent to a Curve